

Laser-modified Angular Distribution of Muon Decay

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We show theoretically that the angular distribution of decay rate of muon can be changed dramatically by embedding the decaying muon in a strong linearly polarized laser field. Evaluating the S -matrix elements taking all electronic multiphoton processes into account. The results suggest the muon may have internal structures instead structureless as in the standard model.

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Introduction.- Recent advances in the generation and control of ultra-intense laser fields [1] paved the way for a number of spectacular applications ranging from particles accelerations [2, 3] and the generation of X -ray pulses [4] to laser-driven nuclear reaction [5], and laboratory astrophysical and high-energy processes [6]. In addition to laser-induced phenomena, modifications of the properties of elementary particles due to the presence of a strong light field are enjoying much of attention. To name but one, Chelkowski *et al.* [7] demonstrated theoretically that upon the ionization and dissociation of muonic molecular ions in superintense laser fields (with intensities $I \sim 10^{21} \text{ Wcm}^{-2}$) the recolliding ions can ignite a nuclear reaction with sub-laser-cycle precision and serve hence as precursors for laser-assisted nuclear processes. A crucial assumption in this kind of studies is that the participating elementary particles are stable during the collision process. The purpose of this work is to point out that the particles field-free life time may change dramatically due to the presence of a strong electromagnetic field (field amplitude $\sim 10^6 \text{ Vcm}^{-1}$). To accomplished such a process in experiment, we may shed a laser beam on the beam of muon. The strong field may also be used to affect the decay of other unstable particles, and reveal new aspects of the decay mechanism. This aspect of the laser-matter interaction is of a prime importance, e.g. when considering strong-field assisted collisions.

Specifically, we consider the modification of the decay life time of muons due to the presence of a strong laser field. As well known, muons have played a crucial role in the development and assessment of the standard model [8] and the field-free muon decay was the first from all leptonic processes to be investigated in full details [9].

The field-free muon lifetime may be modified by a number of factors: E.g., Czarnecki *et al.* [12] investigated the

modifications of the μ^+ lifetime due to muonium (μ^+e^-) formation and other medium effects, whereas Vshivtsev and Éminov [13] studied the influence of a weak field on τ_μ . In recent years, few investigations of the muon or muonium-laser interactions were carried out: Chu *et al.* studied the laser excitation of the muonium $1S-2S$ transition [14] whereas Nagamine *et al.* [15] reported on an ultraslow μ^+ generation upon laser ionization of thermal muonium. Our focus in this work is on the decay of the muon into an electron, a muon-neutrino ν_μ , and an electronic antineutrino $\bar{\nu}_e$ in a strong laser field. Here we note that present-day laser sources produce intensities of 10^{18} Wcm^{-2} or higher in which case the averaged quiver energy of the electron in the laser field may well exceed its rest energy [16] necessitating thus a full relativistic treatment.

Theoretical formulation.- We assume the decay of a muon to occur in the presence of a monochromatic, linearly polarized, spatially homogeneous laser field. The final state electron is treated relativistically. The electron energy ranges from m_e to $m_\mu/2$ where m_e and m_μ are respectively the rest masses of the electron and the muon. The laser is supposed to be switched on adiabatically for a duration considerably longer than τ_μ . The laser intensity is chosen such that pair creation [17] is negligible. The electromagnetic field is described by the classical four-potential (unless otherwise stated, we use natural units in which $\hbar = c = 1$) $A(x) = a \cos(k \cdot x)$ that satisfies the Lorenz condition. The constant four vector $a = (0, \mathcal{E}_0/\omega)$, where \mathcal{E}_0 is the amplitude of the electric field strength of laser. The wave four vector $k = (\omega, \mathbf{k})$ follows from the laser frequency ω and wave number \mathbf{k} .

The S -matrix elements for the laser-assisted μ^- decay reads [9, 11]

$$S_{fi} = -i \frac{G}{\sqrt{2}} \int [\bar{\psi}_{\nu_\mu} \gamma_\lambda (1 - \gamma_5) \psi_\mu] [\bar{\psi}_e \gamma^\lambda (1 - \gamma_5) \psi_{\nu_e}] d^4x. \quad (1)$$

Here $G = (1.16637 \pm 0.00002) \times 10^{-11} \text{MeV}^{-2}$ is the constant of the weak interaction, and \mathbf{x} stands for the spatial coordinates. ψ_μ , ψ_{ν_μ} , ψ_e , and ψ_{ν_e} are respectively wave functions of the muon, the muonic neutrino, the electron, and the electronic antineutrino. The neutrinos are treated as massless particles describable by Dirac spinors [18]; the minor finite-mass effects can be included as done

in Ref.[11]. For the description of the laser-dressed states we note the following. The muon, due to its large mass, is much less influenced by the laser than the electron (for the laser intensities considered here). The state of the electron in the laser field is represented by the Dirac-Volkov function (normalized in a large volume V) [19]. For a linearly polarized field it has the form

$$\psi_e(x) = \left[1 + \frac{e \not{k} \not{a}}{2(k \cdot p)} \cos(k \cdot x) \right] \frac{u_e}{\sqrt{2EV}} \exp \left[-q \cdot x - \frac{e(a \cdot p)}{k \cdot p} \sin(k \cdot x) \right]. \quad (2)$$

where e is the electron charge, p is the four-momentum of electron for laser free, and $q = p - \frac{e^2}{2(k \cdot p)} \bar{A}^2 k = (E, \mathbf{q})$ can be viewed as the (time) averaged four-momentum of the electron in the presence of the laser field, with \bar{A}^2 being the square of the four-potential averaged in a laser cycle.

u_e is a Dirac bispinor representing the free electron and it is normalized as $\bar{u}_e u_e = 2m_e^2$, where \bar{u}_e is the Dirac adjoint of u_e . Inserting Eq.(2) and the wave functions of mesons into Eq.(1) and after some (exact) algebraic manipulations we find the S-matrix to be expressible as

$$S_{fi} = -i \frac{G}{\sqrt{2}} \sqrt{\frac{m_\mu m_e}{E_\mu E}} \frac{1}{2E_{\nu_\mu} 2E_{\nu_e}} \frac{1}{V^2} \sum_l [\bar{u}_{\nu_\mu} \gamma_\lambda (1 - \gamma_5) u_\mu] [\bar{u}_e f^\lambda v_{\nu_e}] \delta(P - q - k_{\nu_\mu} - k_{\nu_e} - lk), \quad (3)$$

where E_μ , E_{ν_μ} , and E_{ν_e} are respectively the energies of μ , ν_μ and $\bar{\nu}_e$. Furthermore, P , k_{ν_μ} and k_{ν_e} are the four-momenta of μ , ν_μ and $\bar{\nu}_e$. u_μ , u_{ν_μ} , and u_{ν_e} are respectively the free Dirac spinors of them. In Eq. (3) l is the number of photons $f^\lambda = (\Delta_0 \gamma^\lambda + \Delta_1 \not{a} \not{k} \gamma^\lambda)(1 - \gamma_5)$, with $\Delta_0 = J_l(D)$ and $\Delta_1 = \frac{l J_l(D)}{2(a \cdot p)}$, where $D = -\frac{a \cdot p}{k \cdot p}$.

Here $J_l(D)$ is a Bessel function of order l . Integrating over the electron angles (Ω) and the energy spectrum (E) of the final state electron we find the decay rate W is determined by the formula [25],

$$\begin{aligned} \frac{dW}{d\Omega} = & \sum_{l=-\infty}^{\infty} \frac{G^2 \pi}{96\pi^5} \int_{m_e + l\omega}^{m_\mu/2 + l\omega} dE \frac{|\mathbf{q}|}{E} \{ \Delta_0^2 [Q^2(P \cdot p) + 2(Q \cdot P)(Q \cdot p)] + 2\Delta_0 \Delta_1 (a \cdot p) [Q^2(k \cdot P) \\ & + 2(Q \cdot k)(Q \cdot P)] - 2\Delta_1 \Delta_0 (k \cdot p) [Q^2(a \cdot P) + 2(Q \cdot a)(Q \cdot P)] - 2\Delta_1^2 a^2 (k \cdot p) [Q^2(k \cdot P) + (Q \cdot k)(Q \cdot P)] \}. \end{aligned} \quad (4)$$

Here we introduced the photon-number-resolved decay rate W_l and the momentum $Q = P - q - lk$.

Results and discussion. - Now we discuss the numerical

result of the laser modified decay rate. The origin of the coordinate system is chosen to be on the muon (before decay), the z-axis is set along the direction of the electric-

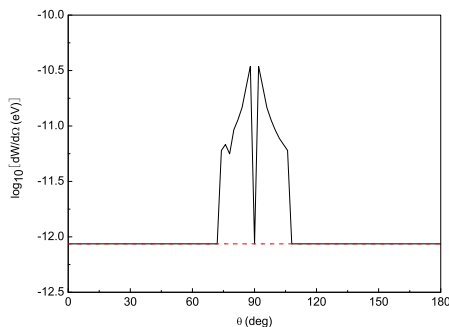


FIG. 1: The muon decay rate versus the polar angle at an azimuth angle $\phi = 90^\circ$ in the presence of a Nd:YAG laser ($\hbar\omega = 1.17 \text{ eV}$) laser with an electric field amplitude of 10^7 V cm^{-1} .

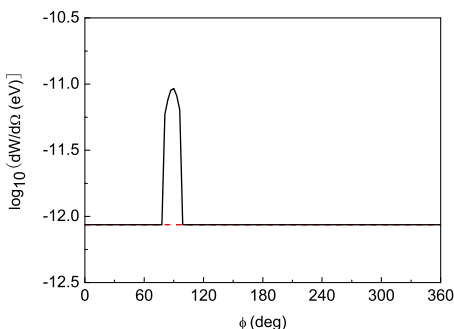


FIG. 2: The muon decay rate versus the azimuth angle at a polar angle $\theta = 80^\circ$. The laser parameters are the same in Fig.1.

field vector ε_0 of the field, and the y -axis is along the direction of the wave vector k .

The numerous sums and integrals involved in the evaluations of W_l have to be performed numerically. The number of contributing multiphoton processes increases rapidly with the field amplitude \mathcal{E}_0 or the intensity $I = \mathcal{E}_0^2/(8\pi)$. This sets a limit on the highest I we are able to consider with our available computational resources. Figs.1 displays the dependence of the decay rate on the polar angle for a Nd:YAG laser ($\hbar\omega = 1.17 \text{ eV}$). It is shown that the laser modification is concentrated in a scope of medium angle around $\theta = 90^\circ$, but close $\theta = 90^\circ$, the laser effect vanishes soon. At other angles, the laser effect vanishes. This suggest that the intermediate meson muon may have internal structure. If the meson has completely no structure as the standard model suggested, the laser-free decay should be isotropic; When the laser is presented, the decay should be enlarged in the polarization direction.

In Fig. 2 we show the decay rate versus the azimuth angle. The decay rate is enlarged around $\theta = 90^\circ$, i.e.

along the direction of the photon momentum of the laser. This suggest the photons are directly interact with the components in the muon, and affect the angular distribution of final state electron.

In summary we provided theoretical evidence that the decay of muon can be modified by a linearly polarized laser field. It suggests muon may have structures, and indicates an approach to study the structure by applying a strong laser background. This effect deserves further investigation both experimentally and theoretically.

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- [1] G. A. Mourou, C. P. J. Barty, and M. D. Perry, Phys. Today **51**, 22 (1998); S.V. Bulanov *et al.*, Phys. Rev. Lett. **91**, 085001 (2003); S. Gordienko *et al.*, Phys. Rev. Lett. **94**, 103903 (2005).
 - [2] R. Snively *et al.*, Phys. Rev. Lett. **85**, 2945 (2000) and references therein.
 - [3] Toma Toncian *et al.*, Science **312**, 410 (2006); B. M. Hegelich *et al.*, Nature **439**, 441 (2006); H. Schwöerer *et al.*, Nature **439**, 445 (2006).
 - [4] A. Rousse *et al.*, Phys. Rev. Lett. **93**, 135005 (2004).
 - [5] K. W. D. Ledingham, P. McKenna, R. P. Shinghal, Science **300**, 1107 (2003); D. Umstadter, Nature (London) **404**, 239 (2000).
 - [6] B. A. Remington, D. Arnet, R. P. Drake, H. Takabe, Science **284**, 1488 (1999); G. A. Mourou, T. Tajima, and S.V. Bulanov, Rev. Mod. Phys. **78**, 309 (2006); M. Marklund and P. K. Shukla, *ibid.* **78**, 591 (2006).
 - [7] S. Chelkowski, A. D. Bandrauk, and P. B. Corkum, Phys. Rev. Lett. **93**, 083602 (2004).
 - [8] David J. Griffiths, *Introduction to Elementary Particles* (John Wiley & Sons, New York, 1987)
 - [9] D. A. Bromley, *Gauge Theory of Weak Interactions* (Springer, Berlin, 2000).
 - [10] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).
 - [11] W. Greiner and B. Müller, *Gauge Theory of the Weak Interaction* (Springer, Berlin, 2003).
 - [12] A. Czarnecki, G. P. Lepage, and W. J. Maraciano, Phys. Rev. D **61**, 073001 (2000).
 - [13] A. S. Vshivtsev and P. A. Éminov, Theo. Math. Phys **44** (2), 284 (1980).
 - [14] S. Chu *et al.*, Phys. Rev. Lett **60**, 101 (1988).
 - [15] K. Nagamine *et al.*, Phys. Rev. Lett **74**, 4811 (1995).
 - [16] H. M. Milchberg and R. R. Freeman, J. Opt. Soc. Am. B **13**, 51 (1996).
 - [17] C. Bula *et al.*, Phys. Rev. Lett. **76**, 3116 (1996).
 - [18] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
 - [19] D. M. Volkov, Z. Phys. **94**, 250 (1935).
 - [20] C. Szymanowski *et al.*, Phys. Rev. A **56**, 3846 (1997).
 - [21] S.-M. Li, J. Berakdar, J. Chen, and Z.-F. Zhou, Phys. Rev. A **67**, 063409 (2003).
 - [22] M. V. Fedorov, *Atomic and Free Electrons in a Strong Light Field* (World Scientific, Singapore, 1997).
 - [23] L. V. Keldysh, Sov. Phys. JETP **20**, 1307 (1965).

- [24] M. Perelomov, V. S. Popov, and M. V. Terentev, Sov. Phys. JETP **23**, 924 (1966).
- [25] We note that Eq.(4) does not account for the influence of radiative corrections which are (in absence of the laser) small in comparison to the modifications brought about by the laser field.